(8 pages) **Reg. No. :** ....

Code No. : 6370 **Sub. Code : ZMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022

Second Semester

Mathematics – Core

ANALYSIS – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Given two partitions, P_1 and P_2. we say that P^* is their common refinement if P^* = ————

   (a) $P_1 \cap P_2$ (b) $P_1 - P_2$

   (c) $P_1 \cup P_2$ (d) $P_1 + P_2$

2. $\int_{-a}^{b} f\, d\alpha$ ———— $\int_{b}^{-b} f\, d\alpha$

   (a) $\geq$ (b) $\leq$

   (c) $=$ (d) $\neq$

3. If γ is Rectifiable then $\wedge(\gamma) <$ ————

   (a) 0 (b) 1

   (c) 2 (d) ∞

4. If x is irrational then the value of $\lim_{m\to\infty} \lim_{n\to\infty} (\cos m!\pi x)^{2n}$ is ————

   (a) 0 (b) 1

   (c) $-\infty$ (d) ∞

5. The value of $\int_0^{2\pi} (\sin n_k x - \sin n_{k+1} x)^2\, dx =$ ————

   (a) 0 (b) π

   (c) 2π (d) ∞

6. If $\{f_n\}$ is uniformly bounded on E if there exists a number M such that $|f_n(x)|$ ————M $(x \in E, n = 1,2,3,\ldots)$

   (a) $<$ (b) $\leq$

   (c) $>$ (d) $\geq$

7. The value of $\lim_{x\to 0} \frac{\log(1+x)}{x}$ is ————

   (a) 1 (b) 0

   (c) e (d) e^x

8. The value of $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$ is ————

(a) 1 (b) 0

(c) e (d) e^x

9. The value of $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$ is ————

(a) $\frac{\pi}{2}$ (b) π

(c) 2π (d) $\sqrt{\pi}$

10. The value of $\Gamma\left(\frac{1}{2}\right)$ is ————

(a) 0 (b) $\frac{1}{2}$

(c) $\sqrt{\pi}$ (d) π

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove if $f \in \mathscr{R}$ on $[a,b]$ and if there is a differentiable function F on $[a,b]$ such that $F' = f$ then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Or

(b) If f is monotonic on $[a,b]$, and α is continuous on $[a,b]$, then prove that $f \in \mathscr{R}(\alpha)$.

12. (a) Suppose K is compact, and Prove that the following:

(i) $\{f_n\}$ is a sequence of continuous functions on K,

(ii) $\{f_n\}$ converges point wise to a continuous function f on K

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, $n = 1,2,3,...$ Then $f_n \to f$ uniformly on K.

Or

(b) If f maps $[a,b]$ into R^k and if $f \in \mathscr{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then Prove that, $|f| \in \mathscr{R}(\alpha)$, and $\left|\int_a^b f\,d\alpha\right| \leq \int_a^b |f|\,d\alpha$.

13. (a) If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3,...,$ and if $\{f_n\}$ is point wise bounded and equi-continuous on K, then Prove that $\{f_n\}$ is uniformly bounded on K.

Or